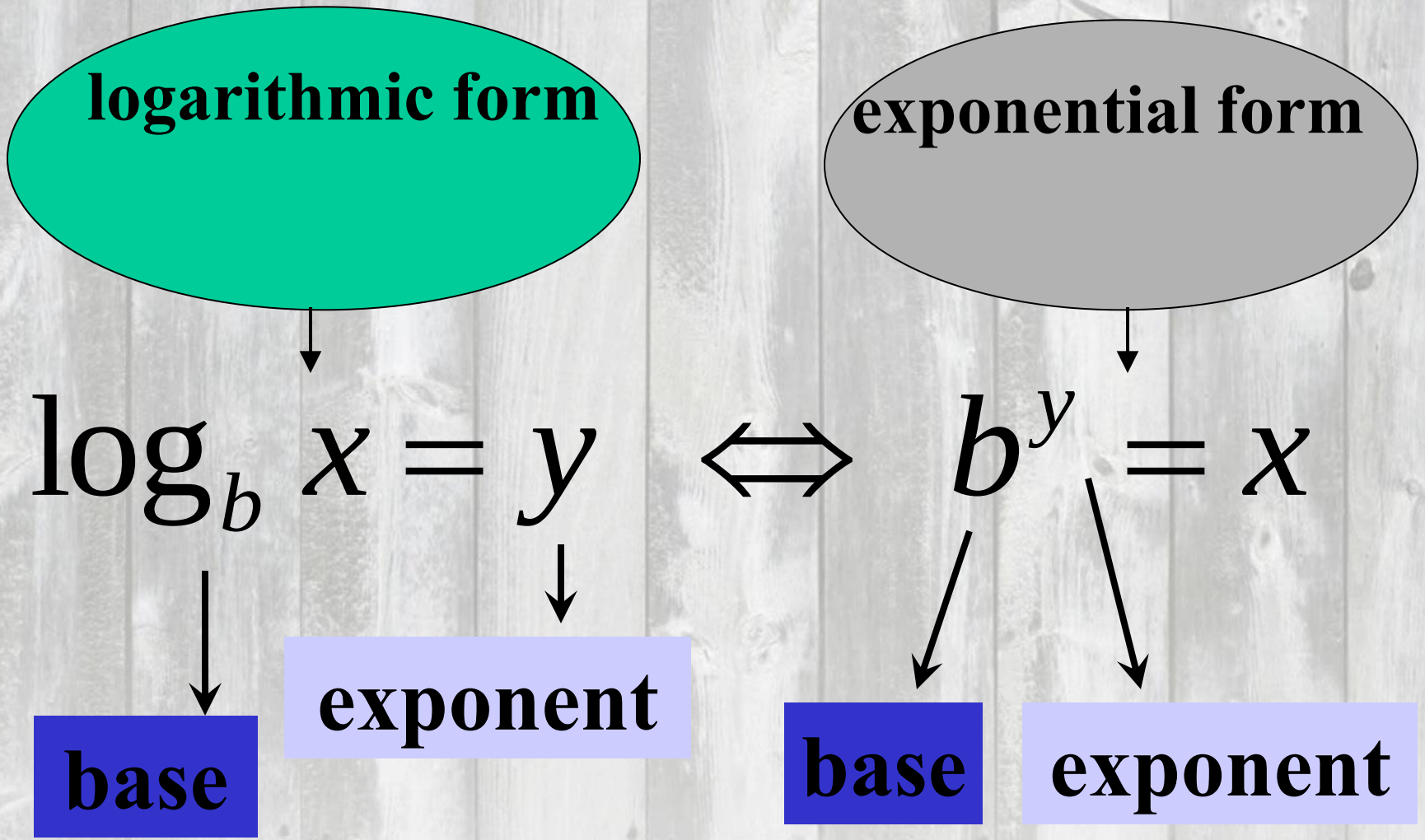
Logarithmic Functions

Objective: You will be able to write the equivalent form of a logarithmic or exponential function as an exponent or logarithm.

The logarithmic funtion to the base b, where $a > 0$ and $a \neq 0$, is denoted by $\log_b x = y$ and is definded by :

$\log_b x = y$ if and only if $b^y = x$



- b is positive with $b \neq 1$

How were logarithms used in the past?

Before there were calculators, mathematicians, scientists, astronomers and engineers used logarithms to multiply, divide and find roots of numbers.

Here's the principal behind using logarithms.

$$16 \times 8 = 128$$

$$\underbrace{2^4 \times 2^3}_{\text{bases are the same, so combine using the laws of exponents}} = 2^{4+3} = 2^7 = 128$$

$$\underbrace{125 \times 81}_{\text{bases are NOT the same, so the laws of exponents do not apply.}}$$

$$\underbrace{5^3 \times 3^4}_{\text{bases are NOT the same, so the laws of exponents do not apply.}}$$

The Past & The Present

$$125 \times 81$$

$$\underbrace{5^3 \times 3^4}$$

bases are NOT the same, so the laws of exponents do not apply.

$$125 \quad \times \quad 81 \quad = \quad 10{,}125$$

$$\underbrace{10^{2.096910} \times 10^{1.908485}}$$

bases are the same, so combine using the laws of exponents.

$$= 10^{2.096910+1.908485}$$

$$= 10^{4.005395}$$

$$= 10{,}124.99926 \approx 10{,}125$$

$$\log 125 \approx 2.096910$$

$$\log 81 \approx 1.908485$$

Logarithm and Exponential Function Equivalent Forms

The equivalent of $y = b^x$ is the function

$$\log_b y = x$$

What if you knew $8192 = 2^x$. How would you find x?

Use the definition of a logarithm: $x = \log_b y$

And with the little help of a calculator we find $\log_2 8192 = x$ is $x = 13$.

Exponential	Logarithmic
1. $a^7 = z$	$\log_a z = 7$
2. $10^2 = 100$	$\log_{10} 100 = 2$
3. $4^3 = 64$	$\log_4 64 = 3$
4. $x^z = y$	$\log_x y = z$

Logarithmic	Exponential
1. $\log_2 32 = 5$	$2^5 = 32$
2. $\log_3 7 = y$	$3^y = 7$
3. $\log_3 a = 7$	$3^7 = \mathrm{a}$
4. $\log_3 9 = 2$	$3^2 = 9$
5. $\log_7 \sqrt{7} = \frac{1}{2}$	$7^{\frac{1}{2}} = \sqrt{7}$
6. $\log_3 \frac{1}{81} = -4$	$3^{-4} = \frac{1}{81}$

The Common Logarithmic Function base =10

$$y = \log_{10} x \quad \Leftrightarrow \quad x = 10^y$$

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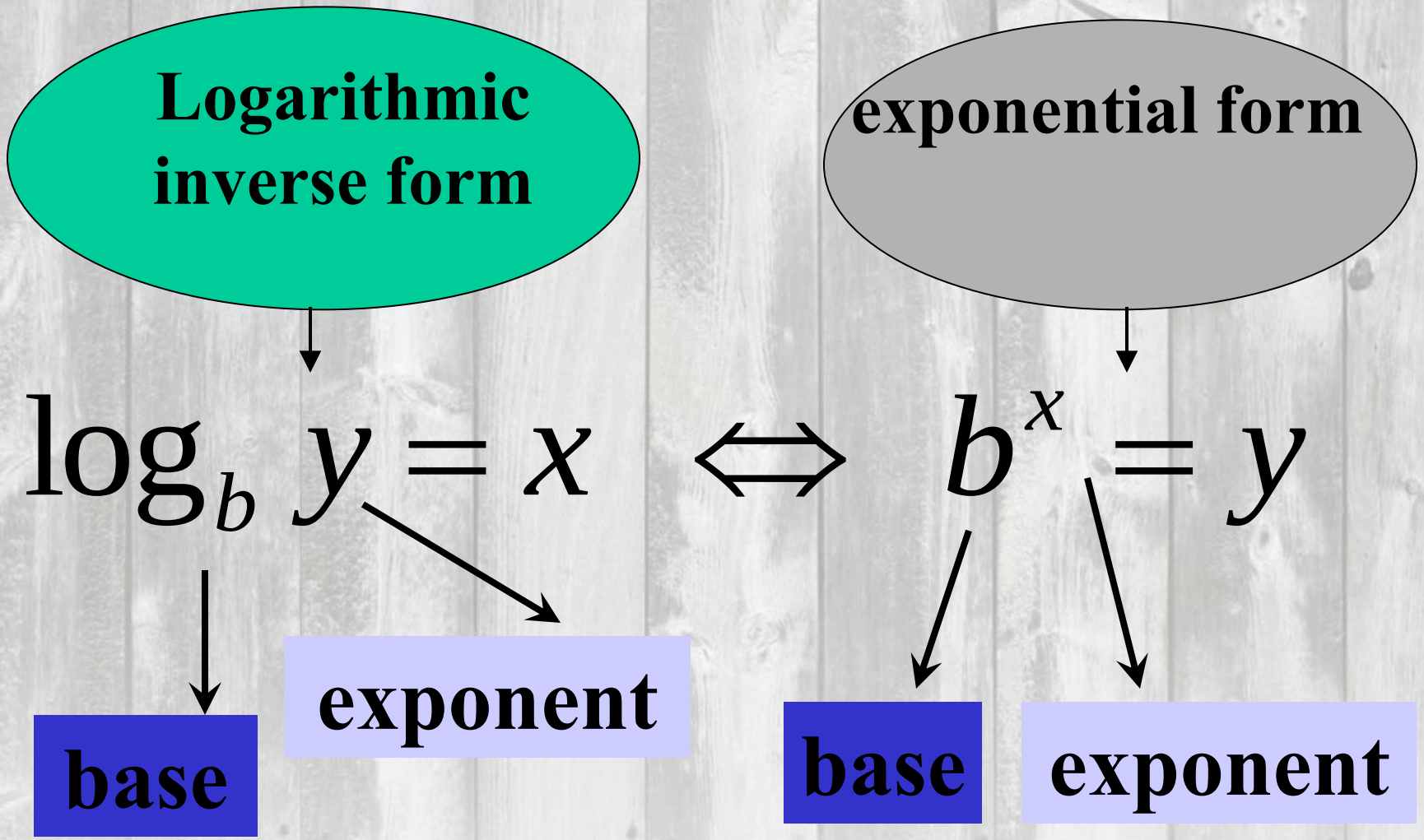

- b is positive with $b \neq 1$